2. The Special Composition Question

In this section I will present a precise formulation of the Special Composition Question. This formulation will require some logical apparatus and some remarks about the theory on which that apparatus rests.

I have said that the Special Composition Question is the question, In what circumstances is a thing a (proper) part of something? But this is a misleading formulation of the question we shall be addressing under that name. This formulation suggests that our question is, In what circumstances does a pair of objects satisfy the predicate '*x* is a proper part of *y*'? But to ask that question at the outset of our inquiry would be to begin *in medias res.* The relation expressed by '*x* is a proper part of *y*' is antisymmetrical, and a pair of objects related by it will in typical cases—in my view, in *all* cases—be very unlike each other. For example: A plank is very unlike a ship; a cell is very unlike a whale. I have found it to be of great heuristic value to let one of these objects (the larger and more complicated) drop out of the picture and to concentrate on the other (the smaller and simpler) and its fellows—the other planks or the other cells. For example, I have found it helpful to ask not 'In what circumstances is a plank a part of a ship?' but, rather, 'In what circumstances do planks compose (add up to, form) something?' When we ask a question of this sort, we are asking a question about the mutual relations that—at least in typical cases—hold among various objects of the same type (among atoms, among planks, among cells, among bricks), relations in virtue of which they are bound together into a whole. If we ever find an answer to a question of this sort, we can go on to ask various questions about what comes to be when atoms or cells or planks or whatever become in this way bound together: We can ask whether its properties supervene upon the properties of and arrangement of its parts; we can ask whether it could survive being taken apart and put together again; we can ask whether it is properly called a ship or a whale. But initially we do not have to talk about it at all, and, I would argue, it is better not to, if only because the fewer the questions a philosopher is attempting to answer simultaneously the better off he is.

Instead of asking about the conditions a pair of objects must satisfy if one is to be a part of the other, therefore, we shall ask about the conditions a plurality (or aggregate, array, group, collection, or multiplicity) of objects must satisfy if they are to compose or add up to something. But I am not entirely satisfied with this formulation of the Special Composition Question either, for I do not like substantives like 'aggregate' and 'plurality'. I do not like them because they are, after all, substantives—and substantives, as it has often been observed, represent themselves as naming substances. Even 'plurality' sounds rather as if, like 'lion' or 'number', it were a name for a kind of object. If we use nouns like 'plurality', 'aggregate', and 'multiplicity', we shall be tempted to ask what the properties of pluralities or aggregates or multiplicities are and how these things achieve their perhaps rather minimal degree of unity and what their identity conditions are. We may even be tempted to ask questions like these: Consider the aggregate of particles that are currently parts of this oak: What is the relationship between the aggregate and the tree? What properties has the one got that the other lacks? Is the tree a temporal succession of aggregates? I think these are bad questions, and I want to avoid them. But even if they were good questions, it would be well to avoid them at the outset of our inquiry, on the principle that recommends dealing with as few questions as possible at any one time. Let us, therefore, place ourselves out of temptation's way.

One way to avoid being tempted by these questions would be to use no collective noun but 'set' in our investigations. It is at least fairly clear that a set is an abstract thing (but many philosophers are puzzled about what that means), and that the existence of a set in no way depends on the relations that hold among its members. Moreover, the laws governing sets are embodied in a much-studied theory of which extremely careful and explicit statements are available. But I think that if we were to conduct our investigation in terms of sets, we should soon discover that the sets played no essential part in our investigation. Suppose we were to ask, In what circumstances does a set of objects compose or add up to something? Unless we take it to be the case that any objects that are metaphysically capable of being parts of something just automatically and of necessity add up to something, we shall, presumably, say that a set of objects adds up to something just in the case that its members bear certain relations (as it may be, causal or spatial) to one another. But then the set itself, as opposed to its members, has dropped out of the picture. If we say, for example, 'The set of blocks on the table adds up to something owing to the fact that the blocks are stacked', the set (as opposed to its members) seems to play no role in our answer beyond that of picking out or enabling us to refer to certain blocks—the ones on the table. But we do not need to appeal to any set to do that. We can call them 'the blocks on the table'. Names of sets are singular terms. Phrases like 'the blocks on the table' are "plural referring expressions." I believe that we can achieve all the powers of plural or collective reference we shall need for our discussion of composition without using singular terms that purport to refer to pluralities or aggregates or sets. We shall need only plural referring expressions.[6]

The idea of a "plural referring expression" has sufficient currency that I need not devote a great deal of space to an explanation of it.[7] 'The members of the college', 'my closest friends', and 'Tom, Dick, and Harry' are plural referring expressions. In my usage, at least, plural referring expressions will not contain quantifier words.[8] (I shall explain this restriction later in the present section.) Thus, in my usage, 'all the king's men' and 'the Mortons and some of the Hanrahans' are not plural referring expressions, though 'the king's men' and 'the Mortons and the Hanrahans' are. I shall assume that we understand "open" plural referring expressions ('Tom, x, and Harry', 'z's men') and the sentences formed by quantifying into them. For example: $\forall x$ (x is a king $\rightarrow \exists y\, y$ and the sons of x are conspiring).

We should note that the word 'and' in 'Tom, Dick, and Harry' and 'the Mortons and the Hanrahans' is not the familiar sentential connective, but a special operator that "takes" singular terms and plural referring expressions and "builds" complex plural referring expressions. The syntax and semantics of this operator (and everything else about it) are so obvious that I shall not bother to discuss it further, nor shall I bother to introduce a notational distinction between 'and' the operator and 'and' the sentential connective.

One may form sentences containing plural referring expressions by combining these expressions with so-called variably polyadic or indefinitely polyadic predicates: 'are in a minority', 'are quarreling', 'are carrying a beam', 'outnumber', 'run the risk of alienating', and so on. We have, for example, such sentences as 'Tom, Dick, and Harry are carrying a beam' and 'The Democrats run the risk of alienating those voters who live on a fixed income'. Each of the predicates 'outnumber' and 'run the risk of alienating' requires both a subject and a direct object, and each is thus in an obvious sense structurally analogous to an ordinary dyadic predicate. The logical structure of variably polyadic predicates is a topic of considerable intrinsic interest, but we shall not need to pursue it.

Some variably polyadic predicates "express" a state or activity that an object can be in or engage in "all by itself." There can be a minority of one, and a man observing two men carrying a beam might say, "I could do that all by myself." (On the other hand, it takes at least two to make a quarrel.) A plural referring expression refers to the objects that satisfy a certain condition, and sometimes this will be a condition that could be satisfied by a single object. Thus, 'my allies' refers to those people who satisfy the condition 'being an ally of mine', and I might have only one ally. Now consider the sentence 'My allies are in a minority'. Suppose I have one ally and six adversaries. I shall count this sentence as expressing, in that circumstance, a truth. In other words, we stipulate that, while the use of a plural referring expression doubtless normally carries the "conversational implicature" that that expression refers to more than one thing, the proposition expressed by a sentence containing a plural referring expression may nevertheless be *true* if that expression refers to only one thing. In consequence of this stipulation, the following sentences express truths: 'Tully and Cicero were Roman orators'; 'The even primes are fewer than the odd primes'; 'The people identical with Frege were all born in the same year'.

We can speak generally about objects without speaking of them collectively. We do this by using the quantifier-variable idiom. Syntactically speaking, variables behave like singular referring expressions (or, to employ the more usual expression, "singular terms"). Is there any linguistic device that stands to plural referring expressions as quantifiers and variables stand to singular terms? Or if there is not, can we invent such a device? The existence of sentences like 'There are some men in the street who are carrying a beam' suggests that there is indeed such a device. In fact, the resources of ordinary English include many such devices, all of them sources of potential confusion and ambiguity. (There are many English adverbs and adverbial phrases whose sole function is to ameliorate this ambiguity. For example: 'individually', 'collectively', 'as a group'. They do not work very well.) We shall need a clear way of writing sentences that stand to sentences like 'Tom, Dick, and Harry are carrying a beam' and 'The Democrats run the risk of alienating those voters who live on a fixed income' as quantificational sentences stand to 'Tom is carrying a beam' and 'The governor runs the risk of alienating my uncle'. I shall not attempt to refine or standardize any of the devices that are used for this purpose in idiomatic English. I shall construct a device for this use out of rather more basic semantical materials.

How are ordinary or "singular" or "individual" variables, the familiar '*x*', '*y*', and '*z*' of the logic texts, introduced? There are many ways of introducing variables. The most satisfactory pedagogically is probably the Berlitz-style method of total immersion favored by mathematics departments. The next-best way stresses the identical syntactical properties of variables and singular terms. The worst way depends on drawing an analogy, or even asserting an identity, between variables and the third-person-singular pronouns of ordinary English. But if one is interested not in introducing variables to beginners in logic or mathematics but in giving an adequate theoretical account of variables to people who already know how to manipulate them, the order is reversed. "Total immersion" is no account at all. Explanations in terms of syntax do not satisfactorily distinguish true variables from dummy or schematic letters. Identifying variables with pronouns, however, provides a genuine explanation of what variables are.

Let us look at the pronominal method, since our interests are theoretical. The *locus classicus* is Quine's *Mathematical Logic.*[9] What follows is inspired by Quine's explanation of variables but does not reproduce it. Imagine a language that is exactly like English except that it has been supplied with an indefinite stock of third-person-singular pronouns, phonetically diverse but semantically indistinguishable. Suppose they are 'it_x', 'it_y', 'it_z', and so on. Call phrases of the form 'it is true of at least one thing that it_x is such that' existential-quantifier phrases. Then introduce the familiar variables and existential quantifier of formal logic by means of the obvious scheme of abbreviation. For example, '$\exists x \exists y\, x$ loves y' will be an abbreviation for

> it is true of at least one thing that it_x is such that it is true of at least one thing that it_y is such that it_x loves it_y.

(A similar account, of course, can be given of the universal quantifier.)

We may follow an exactly parallel route in combining generality with collective reference. Imagine a language that is exactly like English except that it has been supplied with an indefinite stock of third-person-plural pronouns: '$they_x$', '$they_y$', '$they_z$', and so on. (For obvious reasons we shall also need a corresponding stock of objective-case pronouns.) Now abbreviate 'they $(them)_x$' by 'the xs', 'they $(them)_y$' by 'the ys', and so on. Call these phrases *plural variables.* (We shall sometimes find 'those xs' to be stylistically preferable to 'the xs', and we shall therefore count phrases of this form as plural variables.) Phrases of the form

> it is true of certain things that $they_x$ are such that

will be called *existential plural-quantifier* phrases. We shall abbreviate them in a way congruous with our abbreviations of plural variables. For example, the displayed phrase may be abbreviated as 'there are xs such that' or 'there exist xs such that' or 'for some xs'. A worked example:

For some χs and for some ys, those χs run the risk of alienating those **ys**

abbreviates

It is true of certain things that they$_x$ are such that it is true of certain things that they$_y$ are such that they$_x$ run the risk of alienating them$_y$.

A truth-conditional semantics for the plural existential quantifier would be an elaboration of the following statement: A sentence of the form 'For some χs, those χs F' expresses a truth just in the case that there is some (nonempty) set such that the members of that set F.

Universal plural quantification can be introduced in an exactly parallel fashion. Phrases of the form

it is true of any things whatever that they$_x$ are such that

will be called *universal plural-quantifier* phrases and may be abbreviated by phrases of the form 'for any χs'. (For some reason that I can't grasp, it does not seem to be possible to construct plural-quantifier phrases around the words 'all' or 'every'.) The reader may find it instructive to write out the sentence

For any χs, if those χs are finite in number, then there are ys such that those ys are finite in number and there are more of the ys than there are of the χs

in unabbreviated form. A sentence of the form 'For any χs, those χs F' will, of course, express a truth just in the case that every nonempty set is such that its members F.[10]

It is important to realize that 'for some χs' and 'for any χs' do not bind—or interact in any way with—the singular variable 'χ'. By the same token, '$\exists\chi$' and '$\forall\chi$' do not bind, or otherwise interact with, the plural variable 'the χs'. 'The χs' and 'χ' are two distinct variables, just as 'χ' and 'y' are two distinct variables. 'The χs' is, officially at least, a symbol that has no meaningful parts: 'the . . . s' is not a *context* within which a singular variable occurs. (As an aid to clarity, however, I shall not use, for example, 'χ' and 'the χs' in the same sentence.)

The expressive power of a language containing both singular and plural variables and quantifiers is greatly increased by the addition of a variably polyadic operator analogous to the '$\in$' of set theory. I shall use the English words 'is one of' for this operator. (I hope that these words convey its meaning, for no explanation of its meaning is possible.) If everything that is one of the *x*s is one of the *y*s, then we say that the *x*s *are among* the *y*s. Let us write this definition out formally, just to illustrate the syntax of 'is one of':

the *x*s are among the *y*s = df

∀z (z is one of the *x*s → z is one of the *y*s).

If the *x*s are among the *y*s and the *y*s are among the *x*s—if something is one of the *x*s if and only if it is one of the *y*s—then we say that the *x*s *are identical with* the *y*s. If the *x*s are among the *y*s but are not identical with the *y*s, then we say that the *x*s *are properly among* the *y*s.

I said earlier that I should not count as a plural referring expression any expression that contained a quantifier word like 'some' or 'all'. We can now appreciate the reason for this restriction. We obviously want to count as plural referring expressions only those expressions that can replace plural variables in syntactically correct sentences without yielding syntactically incorrect sentences; and 'James is one of some of the Hanrahans' and 'The conspirators are among all of the members of the Senate' are syntactically incorrect. Our official reading of, for example, 'The Mortons and some of the Hanrahans are having a picnic' is 'For some *x*s, the *x*s are among the Hanrahans, and the Mortons and the *x*s are having a picnic'.

Certain plural referring expressions, expressions like 'the Democrats' and 'my closest friends', are in an obvious way analogous to definite descriptions. Having 'is one of' at our disposal, we may, if we wish, provide a "Russellian" treatment of these expressions. For example, we may treat 'The dinosaurs are extinct' as an informal abbreviation for 'For some *x*s [∀*y*(*y* is one of the *x*s ↔ *y* is a dinosaur) & the *x*s are extinct]'.

I shall feel free—even when writing out formal definitions—to use English idioms that do the same work as 'is one of' when their meaning is clear. For example, a formal definition that we shall come to shortly contains the clause 'the *x*s are all parts of *y*'; this is clear and idiomatic and there would be no point in replacing it with an "official" sentence like 'for all z, if z is one of the *x*s, then z is a part of *y*' or 'the *x*s are among the parts of *y*'.

We have, then, open sentences containing both free singular and free plural variables. It is important to distinguish in theory—although not always terribly important to distinguish in practice—between open sentences and *predicates*. The three distinct open sentences '*x* loves *y*', '*y* loves *x*', and '*z* loves *z*', are all instances of a single (dyadic) predicate; and the third, '*z* loves *z*', is *also* an instance of a distinct monadic predicate of which '*x* loves *y*' and '*y* loves *x*' are *not* instances. (But '*x* loves *x*' and '*y* loves *y*' are instances of that predicate.) The same point applies, *mutatis mutandis*, to variably polyadic predicates. I shall not bother to introduce a special notation—involving Greek letters or circled numerals—for predicates. Instead, I shall talk as if predicates contained variables. The reader will always be able to tell whether, when I mention an open sentence in the course of making some point, I mean my point to apply only to *that* sentence—the one containing *those* variables in *those* places—or whether I mean my point to apply to the predicate of which that sentence is a typical instance.

Just as ordinary predicates—predicates containing free singular and no free plural variables—express relations, so variably polyadic predicates—predicates containing free plural variables—express relations. ('Variably polyadic predicate' is not always an appropriate description of a predicate containing a free plural variable; there is nothing "variable" about 'the *x*s are three in number'. I do not think that this will cause any confusion, however.) An *n*-adic predicate (a predicate containing *n* free singular variables and no plural variables) expresses an *n*-ary relation: the dyadic predicate '*x* likes *x* better than *x* likes *y*' expresses a binary relation. Variably polyadic predicates express so-called multigrade relations. For example, 'the *x*s belong to the same political party' expresses a multigrade relation. ('Multigrade relation' is not always an appropriate description of a relation expressed by a predicate containing free plural variables; there is nothing "multigrade" about the relation expressed by 'the *x*s are three in number'. Indeed, it could be plausibly argued that this relation is just the ternary relation expressed by '*x* is not identical with *y* and *y* is not identical with *z* and *x* is not identical with *z*'. If this is so, 'multigrade' and, for instance, 'ternary' are compatible descriptions. But even if this is so, there will be some multigrade relations—such as the one expressed by 'the *x*s belong to the same political party'—that cannot be identified with any *n*-ary relation. Again, I do not think that the traditional terminology will cause any confusion.) If we wished seriously to study multigrade relations, we should have to develop a system of classification analogous to the classification of ordinary relations into unary, binary, ternary (and so on) relations. (Consider, for example, the relation expressed by 'the *x*s and *y* are conspiring against the *z*s, the father of *y*, and the former leader of the *x*s'.) Fortunately, we shall be concerned only with very simple multigrade relations, and we shall be able to get along without a systematic classification of them.

Let us now return to the Special Composition Question. We shall use the expression the xs compose y

as an abbreviation for

the xs are all parts of y and no two of the xs overlap and every part of y overlaps at least one of the xs.

(Remember that we are using 'part' in such a way that everything is a part of itself. A thing *overlaps* a thing—or: they overlap—if they have a common part. If no two of the xs overlap, we shall sometimes say that the xs are *disjoint*.) For example, if there is a house made entirely of bricks, then the bricks compose the house. If, as some philosophers believe, there are such things as the north half and the south half of the house, then the north half and the south half compose the house—and, of course, there is no inconsistency in saying both that the bricks compose the house and that the two halves compose the house.[11] On the other hand, the north half of the house and the south two-thirds of the house do not compose the house, owing to the fact that they have parts in common.

It follows from our definition that, for any x, the things identical with x compose x. If the xs compose y and the xs are two or more, then we shall say that the xs *properly* compose y.

The verb 'compose' in the predicate 'the xs compose y' is to be understood as being in the present tense, and the same point applies to 'are' in 'are parts of' and to all other verbs that occur in the *definiens* of 'the xs compose y'. Thus, 'are parts of' and 'compose' should be read 'are *now* parts of' and '*now* compose'. Strictly speaking (given the use we shall make of it), our *definiendum* should have been 'the xs compose y at t', and our "primitive" mereological predicate should have been 'x is a part of y at t'. But I do not think that any confusion will result from our not speaking that strictly.

In addition to the notion of composition, it will be useful to have the weaker notion of summation, summation being just composition without the "no overlap" requirement:

y is a sum of the xs = df

the xs are all parts of y and every part of y overlaps at least one of the xs.

For example, the north half and the south two-thirds of our house of bricks have the house as a sum; the house is a sum of the house, the north half of the house, and any seven of the bricks. (It will be noted that we say '*a* sum' rather than '*the* sum'. Although *I* think that, for any *x*s, there is at any given time at most one object such that the *x*s are all parts of that object and every part of that object overlaps at least one of the *x*s, I am willing to argue with anyone who thinks otherwise, and I see no reason to define 'sum' in such a way as to unfit it for use in such an argument. And there *are* those who think otherwise. Some philosophers believe that a gold statue and the lump of gold from which it is formed are numerically distinct objects. Such philosophers presumably believe that there are *x*s—certain gold atoms, for example—such that the statue is one sum of the *x*s and the lump is another sum of the *x*s. We shall touch on the question of the lump and the statue in Sections 5 and 13.)

Having defined 'sum', we are in a position to examine the sentence 'A whole is the sum of its parts'. I want to take a moment to do this because many philosophers misuse this sentence. Is it true that "a whole is the sum of its parts"? We have decided not to legislate about the question whether, in general, the *x*s can have more than one sum (at a time). Let us, therefore, examine the thesis that a whole is *a* sum of its parts. If we take a whole to be something that has parts—and what else could a whole be?—this thesis may be formally expressed as follows: If *x* has parts, then *x* is a sum of the parts of *x*'. Now, in our inclusive sense of 'part', according to which *x* is itself one of the parts of *x*, this thesis is obviously a trivial truth. But even the thesis 'if *x* has proper parts, *x* is a sum of the proper parts of *x*' is a trivial truth. This may be seen by substituting 'the proper parts of *y*' for 'the *x*s' in the above definition of 'sum'. Why, then, do some philosophers (philosophers unconcerned with the question whether there could be objects that had more than one sum) persist in treating the sentence 'A whole is the sum of its parts' as if it expressed a substantive metaphysical thesis? The answer is, I think, that they use this sentence to express the proposition that, if *y* is the (or a) sum of the *x*s, then the intrinsic properties of the *x*s, and the relations in which they stand to one another, determine the intrinsic properties of *y*; that the intrinsic properties of a whole *supervene upon* the intrinsic properties and arrangement and causal interactions of its proper parts.[12] This is indeed a substantive metaphysical thesis, but (unless it were made to do so as the consequence of an arbitrary stipulation, a stipulation having no basis in the meanings of the words that make up the sentence) the sentence 'A whole is the sum of its parts' does not express that thesis.[13]

We now have the equipment necessary for a reasonably precise discussion of the Special Composition Question. Our official formulation of the Special Composition Question is this: When is it true that

$\exists y$ the *x*s compose *y*?

More formally, can we find a sentence which contains no mereological terms and in which no variable but 'the χs' is free and which is necessarily extensionally equivalent to '$\exists y$ the χs compose y'? (Two sentences are "necessarily extensionally equivalent" if the universal closure of their biconditional is a necessary truth. A "mereological term" is a word or phrase that can be given a trivial definition in terms of 'part'. For example, 'sum' and 'compose' are mereological terms.) Less formally, in what circumstances do things add up to or compose something? When does unity arise out of plurality?[14]

It will be instructive to approach the Special Composition Question as if it were a practical rather than a theoretical question. Here is a "practical" version of the Special Composition Question:

> Suppose one had certain (nonoverlapping) objects, the χs, at one's disposal; what would one have to do—what *could* one do—to get the χs to compose something?

For example: Suppose that one has a lot of wooden blocks that one may do with as one wills; what must one do to get the blocks to add up to something? Asking the Special Composition Question in this "practical" form has the virtue of concentrating our attention on the χs and on the question, What multigrade relation must the χs (be made to) bear to one another in order for them to form a whole? (I am going to assume that for no *two* relations is it true in every possible world that just the same objects enter into each of those relations in that world. Hence, necessarily extensionally equivalent sentences express the same relation. Nothing substantive hangs on this assumption, which could be dispensed with at the cost of some verbal complication in the sequel.) The question 'In virtue of what do these n blocks compose this house of blocks?' is a question about $n + 1$ objects, one of them radically different from the others. But the question 'What could we do to get these n blocks to compose something?' is a question about n rather similar objects. To adapt what I said about parthood at the beginning of this section to the case of composition, questions of the former sort turn our minds to various metaphysical and linguistic questions about the "special" $n + 1$st object and our words for it: What are the identity conditions for houses of blocks? Is 'house of blocks' a phase-sortal? And so on. It may be that our inquiries will eventually force us to attend to such questions, but we can make a good beginning without raising them.

The present section has been devoted to various logical points about plurality and composition. There are certain other logical points related to these notions that will need to be made explicit. But, although these points are very general and are no more closely tied to one answer to the

Special Composition Question than to another, they will be more easily grasped if they are presented in connection with a fairly detailed exposition of a theory about the conditions under which composition occurs that they would be if they were presented in the abstract. I therefore postpone discussion of them till we have examined an answer to the Special Composition Question.[15]